Modulation Compression for Short Wavelength Harmonic Generation

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Laser modulator is used to seed free electron lasers. In this paper, we propose a scheme to compress the initial laser modulation in the longitudinal phase space by using two opposite sign bunch compressors and two opposite sign energy chirpers. This scheme could potentially reduce the initial modulation wavelength by a factor of C and increase the energy modulation amplitude by a factor of C, where C is the compression factor of the first bunch compressor. Such a compressed energy modulation can be directly used to generate short wavelength current modulation with a large bunching factor.

The tunable short wavelength free electron lasers (FELs) provide great opportunities for scientific discoveries in biology, chemistry, material science and physics, and form a basis for fourth generation light source. Currently, there are growing interests in using high-gain harmonic generation (HGHG) scheme to generate such short wavelength radiation [1, 2]. The HGHG scheme can produce an FEL radiation with a good transverse and longitudinal coherence, high stability and shorter saturation length compared with the self-amplified spontaneous emission approach. The standard single stage HGHG scheme consists of a modulator that converts the laser field oscillation into an electron beam energy modulation through laser-electron resonant interaction inside an undulator, a dispersive element that converts the electron beam energy modulation into current density modulation, and an FEL radiator that produces coherent radiation using a harmonic of electron beam current modulation. The efficiency of this up-frequency conversion is limited by the amplitude of the harmonic component (bunching factor) inside the electron beam current modulation that decreases exponentially with the increase of the harmonic number. This limits the harmonic number usually to a small number. Several methods were proposed to improve the up-frequency conversion efficiency of the single stage HGHG with some modest success [3, 4]. Recently, a new approach based on beam echo effect that can significantly improve the up-frequency conversion efficiency was proposed for generation of short wavelength radiation [5]. The bunching factor of the high harmonic in the echo enabled harmonic generation (EEHG) scheme decreases only as cubic root of the harmonic number. This enables the EEHG scheme to generate high harmonic current modulation with a reasonable bunching factor.

Both the HGHG and the EEHG require the use of significant amount of laser power to seed the electron beam. In this letter, we propose a new scheme for generation short wave length radiation by modulation compression. Through a modulation compression, the initial energy modulation amplitude can be amplified by a factor of compression factor while the modulation wavelength is reduced by a factor of compression factor. This significantly reduces the laser power needed for generation of short wavelength radiation. Meanwhile, since the modulation wave length has already been reduced by a factor of the compression factor, a fundamental harmonic mode of such modulation can be used directly to produce short wave length current modulation. This results in a large bunching factor for such a short wave length modulation. Furthermore, using a few cycle laser pulse modulation, such a modulation compressed beam might also be used to generate atto-second radiation.

A schematic plot of the accelerator beam line element for the proposed scheme is shown in Figure 1. It consists of a laser modulator, an energy chirper A, a bunch compressor A, another energy chirper B, and another bunch compressor B. In this scheme, an initial laser modulated electron beam is transported through an energy chirper to obtain a negative energy chirping across the beam bunch length. This chirped beam is sent into a standard bunch compressor for bunch compressing. After the bunch compressor, the beam is transported through another energy chirper with opposite sign of energy-bunch length correlation compared with that of the first energy chirper. This chirper undoes the global negative energy chirp across the beam and generates a local positive energy chirp with an amplified energy modulation amplitude. After the second energy chirper, the beam is transported through the second bunch compressor that has opposite sign of R_{56} compared with the first bunch compressor. This bunch compressor further compresses the positive chirped beam and results in a compressed energy modulation with increased modulation amplitude and reduced modulation wave length. Some similar schemes were proposed by Biedron et al. and Shaftan et al. for tuning HGHG radiation wavelength [6, 7]. A key difference between the scheme proposed in this Letter and the previous schemes is the usage of the second opposite sign bunch compressor. Without such a bunch compressor, the initial modulation of the beam will not be further compressed while the original modulation structure inside the locally chirped beam is smeared out by the finite energy spread as shown in the latter part of this Letter.

In the following, we will derive the longitudinal phase space distribution of the beam trans-

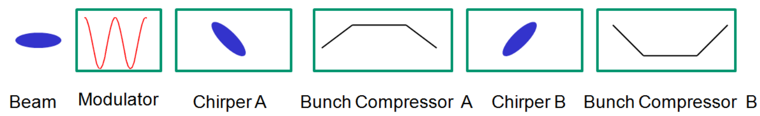

FIG. 1: A schematic plot of the accelerator lattice for modulation compression.

porting through above ideal accelerator lattice using a one-dimensional model. The beam is assumed to be longitudinal frozen in most part of the lattice except that in the bunch compressors. The initial longitudinal phase space distribution function before the laser modulator is given as, $f_0(z_0, \delta_0) = F(z_0, \delta_0/\sigma)$, where z_0 is the longitudinal bunch length, $\delta_0 = \Delta E_0/E_0$ is the relative energy deviation, σ is a constant related to the initial energy spread. After the laser modulator, the energy deviation becomes, $\delta_1 = \delta_0 + A\sin(kz_0)$, where $A = V_1/E_1$ is the initial laser modulation amplitude, $\delta_1 = \Delta E_1/E_1$, $E_1 = E_0$, and k is the modulation wave number. Now, the beam is transported through the chirper A that will introduce an energy-bunch length correlation, $\delta_2 = D^a\delta_1 + h^a z_1$, where $h^a = d\delta_2/dz_2$ is the energy chirp across the bunch length of the beam, $\delta_2 = \Delta E_2/E_2$, and $D^a = E_1/E_2$ denotes the ratio of total beam energy before and after the chirper A. Next, the beam passes through the bunch compressor A. The longitudinal bunch length becomes, $z_3 = z_2 + R^a_{56}\delta_2$, where R^a_{56} is the momentum compaction factor of the bunch compressor A. After the bunch compressor, the phase space distribution becomes

$$f_3(z_3, \delta_3) = \frac{1}{D^a}F(z_3 - R^a_{56}\delta_3, \frac{\delta_3 - h^a C z_3 - CD^a A\sin(k(z_3 - R^a_{56}\delta_3))}{CD^a\sigma}) \quad (1)$$

where $C = 1/(1 + R^a_{56}h^a)$ is the bunch compression factor of the first bunch compressor A. Then the beam is transported through the chirper B that will introduce another energy-bunch length correlation, $\delta_4 = D^b\delta_3 + h^b z_3$, where h^b is the energy chirp across the bunch length of the beam caused by the second chirper, $D^b = E_3/E_4$ denotes the ratio of total beam energy before and after the chirper B, and $\delta_4 = \Delta E_4/E_4$. The phase space distribution becomes

$$\begin{aligned} f_4(z_4, \delta_4) &= F(z_4(D^b + h^b R^a_{56})/D^b - R^a_{56}/D^b\delta_4, \\ &\quad \frac{\delta_4 - (h^b + h^a D^b C)z_4 - CD^bD^a A\sin(k(z_4(D^b + R^a_{56}h^b)/D^b - R^a_{56}/D^b\delta_4))}{CD^bD^a\sigma}) \quad (2) \end{aligned}$$

If the second chirper is set up so that $h^b = -h^a D^b C$, then the distribution function can be written as

$$f_4(z_4, \delta_4) = \frac{1}{D^aD^b}F(Cz_4 - R^a_{56}/D^b\delta_4, \frac{\delta_4 - CD^bD^a A\sin(kCz_4 - kR^a_{56}/D^b\delta_4)}{CD^bD^a\sigma}) \quad (3)$$

Finally the beam is transported through the second bunch compressor B. The longitudinal bunch length of the beam becomes, $z_5 = z_4 + R^b_{56}\delta_4$, where R^b_{56} is the momentum compaction factor of the bunch compressor B. The final longitudinal phase space distribution becomes

$$f_5(z_5,\delta_5) \;=\; \frac{1}{D^aD^b}F(Cz_5-(R_{56}^b+R_{56}^a/D^b)\delta_5, \frac{\delta_5-CD^bD^aA\sin(kCz_5-(kCR_{56}^b+kR_{56}^a/D^b)\delta_5)}{CD^bD^a\sigma}) \tag{4}$$

If the second bunch compressor is set up so that $R_{56}^b = -R_{56}^a/(D^bC)$, then the final longitudinal distribution function can be written as

$$f_5(z_5,\delta_5) \;=\; \frac{1}{D^aD^b}F(Cz_5, \frac{\delta_5-CD^bD^aA\sin(kCz_5)}{CD^bD^a\sigma}) \tag{5}$$

From the final longitudinal distribution f_5, we see that the modulation wave length is reduced by a factor of C and the relative amplitude of modulation is increased by a factor of D^bD^aC. Writing the final energy modulation amplitude in absolute unit of energy, this results in a final energy modulation amplitude CV_1 (eV), where V_1 is the initial absolute energy modulation amplitude due to the laser beam interaction inside the modulator.

As a numerical illustration, we assume $D^a = D^b = 1$ (i.e. no acceleration through the proposed accelerator lattice), initial uniform current distribution and Gaussian energy distribution with a relative energy spread $\sigma = 0.5\times10^{-4}$, initial relative modulation amplitude $A = 2\times10^{-4}$, initial laser modulation wavelength 1 um, the first chirp $h^a = -19$ (m^{-1}), the first bunch compressor $R_{56}^a = 5$ cm, the second chirp $h^b = 380$ (m^{-1}), and the second bunch compressor $R_{56}^b = -2.5$ mm. This will result in a compression factor 20 from the first bunch compressor. Figure 2 shows the longitudinal phase space of the beam after the initial modulation, after the second chirper B, and after the second bunch compressor B. It is seen that after the second chirper, even though the global energy chirp is removed, there still exists significant local positive chirp. The modulation structure in the local chired beam is smeared out by the initial energy spread. After the second bunch compressor, the beam is further compressed. This restores the initial energy modulation structure as shown in the last plot of the Figure 2. The final modulation wave length is reduced from the initial 1 um down to 0.05 um while the modulation amplitude increases from 3.5×10^{-4} to 7.0×10^{-3} as expected.

The above compressed energy modulation will not generate current density modulation. However, by tuning the R_{56}^b of the second bunch compressor, a strong current density modulation with reduced wave length will appear. This is similar to the standard process of high harmonic generation. Assume an initial uniform current distribution with a Gaussian energy distribution, and $R_{56}^b = -R_{56}^a/(D^bC)+\Delta r^b$, then the distribution function after the bunch compressor B will be

$$f(z,\delta) \;=\; \exp(-\frac{1}{2}(\frac{\delta-CD^bD^aA\sin(kCz-kC\Delta r^b\delta)}{CD^bD^a\sigma})^2) \tag{6}$$

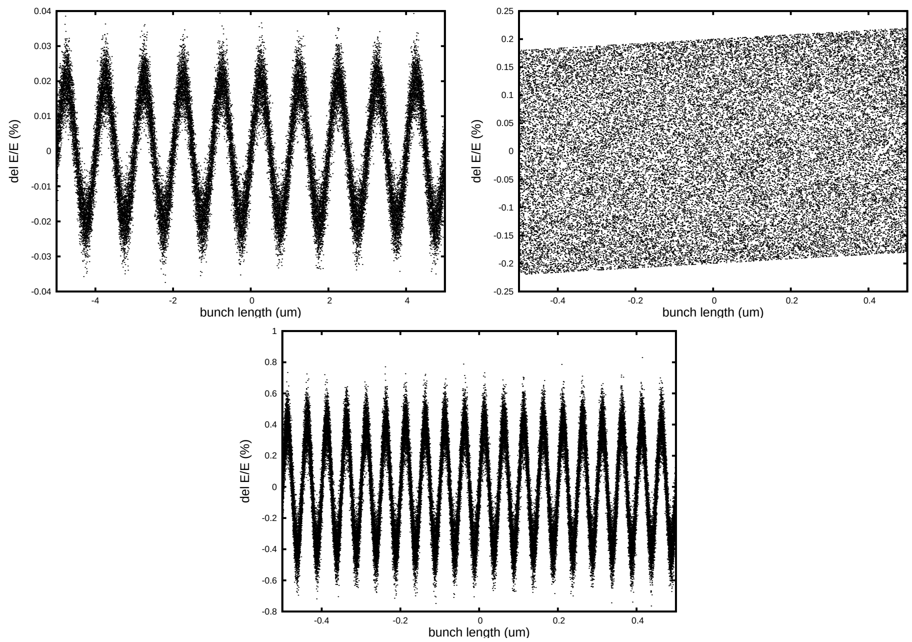

FIG. 2: Longitudinal phase space evolution after initial modulation (left), after the second chirper (middle), and after the second bunch compressor (right).

The current density distribution $I(z)$ can be found from integration of the above distribution over δ. This integral has the same form as that in the standard high harmonic generation scheme [2]. It is a periodic function of z and can be represented as a Fourier series yielding [9]:

$$I(z) \ = \ I_0(1 + 2\sum_{n=1}^{\infty} b_n \cos(nCkz)) \tag{7}$$

where the coefficient b_n (i.e. bunching factor of harmonic n) is given as

$$b_n \ = \ J_n(nC^2k\Delta r^b AD^bD^a)\exp(-\frac{1}{2}n^2(C^2k\Delta r^b D^b D^a\sigma)^2) \tag{8}$$

where the J_n is the Bessel function of order n. From above equation, it is seen that the bunching factor has a strong dependence on the harmonic number. Since the initial modulation wavelength has already been compressed by a factor of C, it might be sufficient to use fundamental harmonic ($n = 1$) for short wavelength FEL radiation. Following the concept of high harmonic generation, we can also optimize the design for higher harmonic number. This might result in an even shorter wavelength radiation with wavelength $\lambda/(nC)$, where λ is the seed laser wavelength. From Eq. 8, we can see that the bunching factor also depends strongly on the products of compression factor of the first bunch compressor and the Δr^b of the second bunch compressor. Without using the second bunch compressor, i.e. $R_{56}^b = 0$ or $\Delta r^b = R_{56}^a/(D^bC)$, with a compression factor of 20 from the first bunch compressor in above numerical example, this will result in a very large exponential decreasing factor (10^{-49348}) in the bunching factor of the initial 1 um wavelength modulation even for the fundamental harmonic. In order to maximize the bunching factor for high harmonic generation, the R_{56}^b of the second bunch compressor needs to be carefully chosen so that Δr^b is sufficiently small to minimize the exponentially decreasing factor in Eq. 8 while maximizing the Bessel function contribution. As an illustration, using the preceding numerical example, we set the R_{56}^b of the second bunch compressor 0.1% off the matched value. Figure 3 shows the current density distribution and the bunching factor of the beam after the second bunch compressor B with slight offset of the R_{56}^b. Very strong current modulation at the wave length of 0.05 um is observed in the left plot of the Figure 3. From the right plot of the Figure 3, we see that there also exists significant extent of bunching factor even for 4th order harmonic that corresponds to a wavelength of 12.5 nm.

Significant amount of energy chirp is needed in the proposed scheme. When a beam pass through a section RF linac, the energy chirp factor h across the beam is given by

$$h = \frac{eV}{E} k_{rf} \sin(\phi) \tag{9}$$

where V is the maximium voltage through the chirper, E is the final energy of the beam, k_{rf} is wave number of wave inside the RF cavity, ϕ is the synchronous phase between the electron beam and the RF wave. At a fixed energy (i.e. $\phi = \pi/2$), in order to achieve a large energy chirp, a shorter wave length cavity will be better. For a beam with 250 MeV energy, passing through an RF linac of 1.3 GHz, it requires 174.5 MeV to generate an energy chirp of 19 (m^{-1}). Assume an average acceleration gradient 10 (MeV/m), it will need an RF accelerating structure of about 20 meters. To generate an energy chirp of 380 (m^{-1}) for the chirper B, this will correspond to 400 meter accelerating structure. However, if we use a higher RF frequency cavity such as 3.9 GHz RF cavity, the length of this chirper can be reduced by a factor of 3 assuming the same average accelerating gradient. Using an X-band RF cavity, e.g. 30 GHz with 150 MeV/m average accelerating gradient [8], it can be reduced to about 1 meter. Another method to generate such a large energy chirp is to use a long wave length laser to interact with the beam inside an undulator if the bunch length of the beam after the first bunch compressor is shorter than the laser wave length. Furthermore, the longitudinal short-range structure wake field can also be used to generate a positive chirp of the beam.

Collective effects from the interactions of electrons can distort the phase space distribution and smear out the modulation structure inside the beam when the beam is transported through the accelerator lattice. On a long wave length scale, the longitudinal space-charge and wake field effects

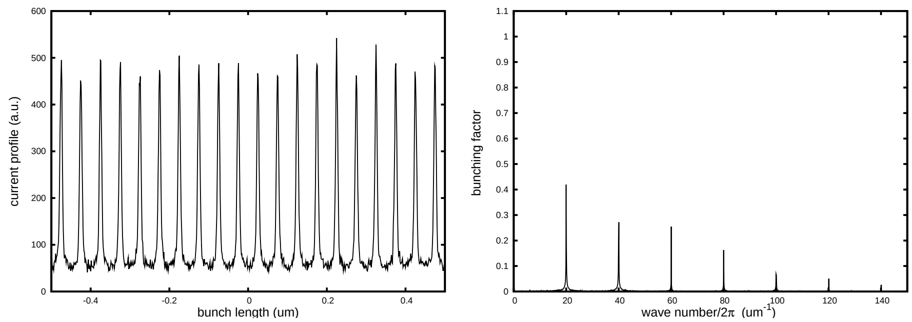

FIG. 3: Current density profile (left) and bunching factor (right) of the beam after the second bunch compressor with detuned R_{56}.

could cause an energy-bunch length chirp across the beam. For a beam with a flat top current distribution which is desired for FEL radiation application, this nonlinearity is mainly near the edge of the beam. Both the longitudinal space-charge field and the structure wake field provide positive chirp to the beam. The positive energy-bunch length correlation from those fields will actually help unchirp the beam after the first bunch compressor and reduce the requirement of chirping for the second energy chirper. Besides the effects on a global long wave scale, the longitudinal space-charge field and the wake field can also drive local modulation instability, i.e. microbunching instability [9–11]. However, previous studies of the microbunching instability showed that the gain of instability will be significantly suppressed for short wavelength modulation due to the finite transverse emittance and uncorrelated energy spread [10, 11].

The other collective effects from the coherent and incoherent synchrotron radiations inside the bunch compressor may also distort the modulation structure inside the beam. From the previous EEHG related studies, these effects might not be a serious problem [12]. However, the local uncorrelated energy spread caused by all of those collective effects will eventually put a limit on the minimum wavelength that can be achieved for a given beam energy and current. More study will be carried out to address those effects in the future.

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